

Important Theorems and Definitions concerning Derivatives

The Definition of the Derivative of a function at a point $(c, f(c))$	$f'(c) = \lim_{\Delta x \rightarrow 0} \left(\frac{f(c + \Delta x) - f(c)}{\Delta x} \right)$ The Alternative definition is $f'(c) = \lim_{x \rightarrow c} \left(\frac{f(x) - f(c)}{x - c} \right)$
Definition of the Derivative of a Function	$f'(x) = \lim_{\Delta x \rightarrow 0} \left(\frac{f(x + \Delta x) - f(x)}{\Delta x} \right)$ The Alternative definition is $f'(x) = \lim_{z \rightarrow x} \left(\frac{f(z) - f(x)}{z - x} \right)$
Notations for derivatives and higher order derivatives	$f'(x) \quad \frac{d}{dx} f(x) \quad D_x f(x) \quad y' \quad \frac{dy}{dx} \quad D_x y$ $f''(x) \quad \frac{d^2}{dx^2} f(x) \quad D_{x^2} f(x) \quad y'' \quad \frac{d^2 y}{dx^2} \quad D_{x^2} y$
Differentiability Implies Continuity	If a function is differentiable at $x = c$ then the function is continuous at $x = c$.
Causes of Non-differentiability	1. Discontinuity 2. Sharp turn or cusp 3. Vertical tangent
Basic Differential Rules	1. Constant Rule: $D_x [c] = 0$ where c is a constant 2. Power Rule: $D_x (x^n) = nx^{n-1}$ 3. Scalar Rule: $D_x (kf(x)) = kf'(x)$ 4. Sum or difference law $D_x [f(x) \pm g(x)] = f'(x) \pm g'(x)$ 5. Product Law $D_x (f(x)g(x)) = f'(x)g(x) + g'(x)f(x)$ 6. Quotient Law $D_x \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - g'(x)f(x)}{(g(x))^2}$

The Derivatives of Trig functions	$1. D_x(\sin x) = \cos x \qquad 2. D_x(\cos x) = -\sin x$ $3. D_x(\tan x) = \sec^2 x \qquad 4. D_x(\cot x) = -\csc^2 x$ $5. D_x(\sec x) = \sec x \tan x \qquad 6. D_x(\csc x) = -\csc x \cot x$
The Chain Rule	If $y = f(u)$ is a differential function of u and $u = g(x)$ is a differential function of x, then $y = f(g(x))$ is a differential function of x and $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \quad (\text{Leibniz notation})$ or $\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x) \quad (\text{Newton's notation})$
Derivative Rules of logarithms and exponential functions	$\frac{d}{dx} \ln x = \frac{1}{x} \qquad \frac{d}{dx} \log_a x = \frac{1}{x \ln a}$ $\frac{d}{dx} e^x = e^x \qquad \frac{d}{dx} a^x = a^x \ln a$
Derivatives of Inverse Trig Functions	$1. \frac{d}{dx} \text{Arc sin } x = \frac{1}{\sqrt{1-x^2}} \qquad 2. \frac{d}{dx} \text{Arc cos } x = \frac{-1}{\sqrt{1-x^2}}$ $3. \frac{d}{dx} \text{Arc tan } x = \frac{1}{1+x^2} \qquad 4. \frac{d}{dx} \text{Arc cot } x = \frac{-1}{1+x^2}$ $5. \frac{d}{dx} \text{Arc sec } x = \frac{1}{ x \sqrt{x^2-1}} \qquad 6. \frac{d}{dx} \text{Arc csc } x = \frac{-1}{ x \sqrt{x^2-1}}$
The Derivative of an Inverse Function	If $g(x)$ is the inverse function of $f(x)$ and $g(b) = a$ then $g'(b) = \frac{1}{f'(a)}$