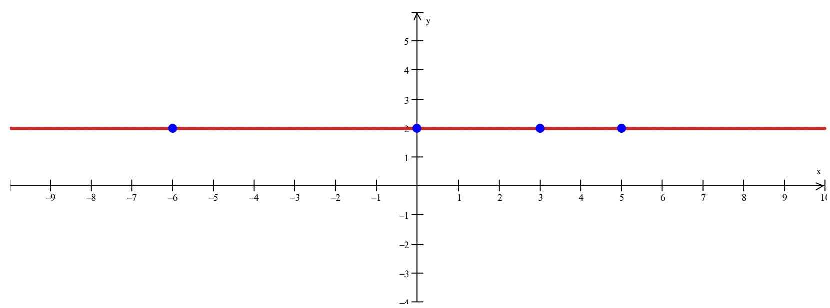
Linear Equations

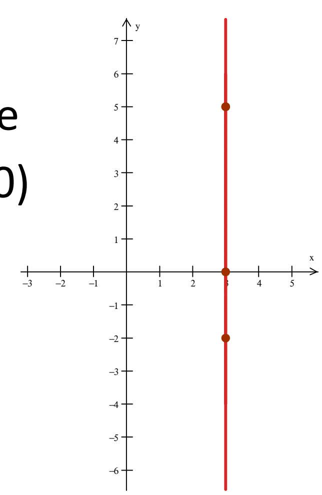
There are three main types of linear equations: horizontal lines, vertical lines and oblique lines.

a) Horizontal lines (y=k)

Horizontal lines contain points that all have the same ordinate or y-coordinate (ex. (3,2), (5,2), (-6,2) and (0,2) all have the same y-coordinate of 2. The slope of a horizontal line is 0. The equation of a horizontal line is y = k where k is the common y-coordinate. For example, the equation of the line through the points listed above is the line y = 2.



b) Vertical lines (x = h)



Vertical lines contain points that all have the same abscissa or x-coordinate (ex. (3,2), (3,5), (3,-2) and (3,0) all have the same X-coordinate of 3. The slope of a vertical line is undefined. The equation of a vertical line is x = h where h is the common x-coordinate. For example, the equation of the line through the points listed above is the line x = 3.

c) Oblique lines

Oblique lines are non-vertical, non-horizontal lines. To understand oblique lines, you must first understand slope. The formula to find the slope of a line is as follows: the slope m between points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$ provided $x_2 \neq x_1$. If $x_2 = x_1$, then the line is vertical (see Vertical lines in part b).

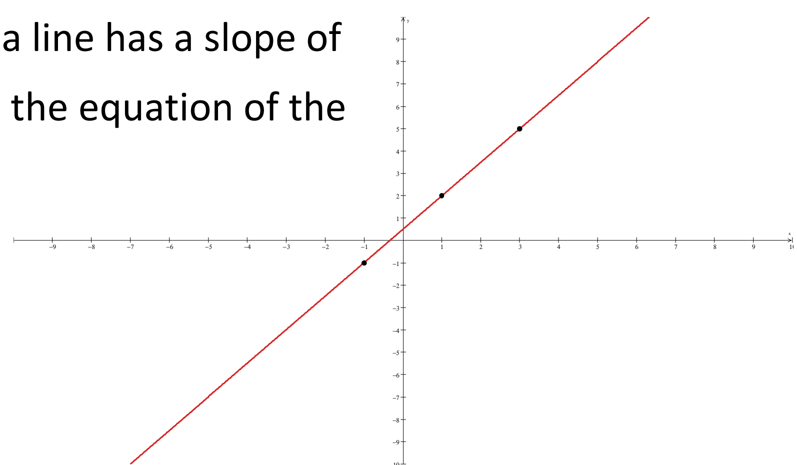
Example, the slope through the line that contains the points (3,5) and (1,2) is $m = \frac{5-2}{3-1} = \frac{3}{2}$ or $m = \frac{2-5}{1-3} = \frac{-3}{-2} = \frac{3}{2}$

Slope is often referred to as $\frac{rise}{run}$ because in the example below, it you start at the point (1,2), you would rise 3 units and run 2 units.

Now consider the line that has the point (x_1, y_1) and the slope m. Since any two points on the line will give the same slope when inserted into the slope formula, then if we let (x, y) stand for any point on the line, then one equation that describes almost all of the points on the line could be $m = \frac{y - y_1}{x - x_1}$. The problem with that equation of the line is that one particular point on the line does not satisfy the equation - namely the one point we were given (x_1, y_1) since evaluating the equation at that point will give us $m = \frac{y_1 - y_1}{x_1 - x_1} = \frac{0}{0}$

To remedy that problem, we simply multiply both sides of the equation by the denominator $x - x_1$ and we obtain the equation:

$y - y_1 = m(x - x_1)$. We call this equation of a line the Point-Slope form of a line, since all it requires is that you know the a point on the line and the slope. For example, if a line has a slope of $\frac{3}{2}$ and contains the point (3,5), then the equation of the line is $y - 5 = \frac{3}{2}(x - 3)$.



Next, let's take the equation of a line that is in Point-Slope form, and solve for y

$$y - y_1 = m(x - x_1)$$

$$y - y_1 = mx - mx_1$$

$$y - y_1 + y_1 = mx - mx_1 + y_1$$

$$y = mx - mx_1 + y_1$$

$$y = mx + b \quad (where\ b = -mx_1 + y_1)$$

$y = mx + b$ is called Slope-Intercept form of a line, the only two numbers in the equation are m which is the slope and b which is the y-coordinate of the y-intercept (0, b).

For example, given the same line from before which has a slope of $\frac{3}{2}$ and contains the point (3,5), then the equation of the line is $y - 5 = \frac{3}{2}(x - 3)$ in point slope form. Solving the equation for y gives us the following steps:

$$y - 5 = \frac{3}{2}(x - 3)$$

$$y - 5 = \frac{3}{2}x - \frac{9}{2}$$

$$y - 5 + 5 = \frac{3}{2}x - \frac{9}{2} + 5$$

$$y = \frac{3}{2}x - \frac{9}{2} + 5$$

$$y = \frac{3}{2}x - \frac{9}{2} + \frac{10}{2}$$

$$y = \frac{3}{2}x + \frac{1}{2}$$

Finally, there is another popular form of a line called Standard Form of a line. A line is in Standard Form, when it is written in the form

Ax + By = C, where A, B, and C are integers (no fractions and no decimals) and A > 0.

A similar form of the line is the General Form Ax + By + C = 0 and some books will refer to this form as Standard.

Continuing or example from before:

$$y = \frac{3}{2}x + \frac{1}{2}$$

$$2y = 2\left(\frac{3}{2}x + \frac{1}{2}\right)$$

$$2y = 3x + 1$$

$$-3x + 2y = -3x + 3x + 1$$

$$-3x + 2y = 1$$

$$-1(-3x + 2y) = -1(1)$$

$$3x - 2y = -1$$

One of the main advantages to having a line in Standard form is that it is very easy to find the x-intercept and the y-intercept (the points where the line crosses the x-axis and the y-axis). Since all points that lie on the x-axis has 0 for a y-coordinate, simply replace y with 0 and solve for x to find the x-intercept. And since any point that lies on the y-axis has 0 for a x-coordinate, replace x with 0 and solve for y.

To find the x-intecept Ax + By = C Ax+B(0) = C (let y = 0) Ax = C x = $\frac{C}{A}$ So the x-intercept is $\left(\frac{C}{A}, 0\right)$	To find the y-intercept Ax + By = C A(0) + By = C (let x = 0) By = C Y = $\frac{C}{B}$ So the y-intercept is $\left(0, \frac{C}{B}\right)$
Using the example $2x + 2y = -7$	
To find the x-intecept $3x - 2y = -1$ $3x - 2(0) = -1$ (let y = 0) 3x = -1 x = $\frac{-1}{3}$ So the x-intercept is $\left(\frac{-1}{3}, 0\right)$	To find the y-intercept $3x - 2y = -1$ $3(0) - 2y = -1$ (let x = 0) $-2y = -1$ Y = $\frac{1}{2}$ So the y-intercept is $\left(0, \frac{1}{2}\right)$